Dynamic Modeling and Adaptive Control of a Single-Link Flexible Manipulator

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This paper presents an application of model reference adaptive control to the position and vibration control of a single-link flexible manipulator. A distributed mass model of the flexible link has been analyzed using the method of modal expansion. The performance of the system with various control schemes has been investigated. It has been demonstrated that effective position/vibration control of a flexible manipulator is achieved by model reference adaptive control.

Introduction

IN the last 40 years industry has seen an ever-increasing trend toward automation. And this trend seems only to be continuing, if not increasing. The last decade has seen robots and manipulators beginning to contribute significantly toward this automation. Robots are now being implemented on a large scale in some industries. Unfortunately, there are still problems that have to be overcome. One of the major problems related to robots is their excessive weight.

The positional accuracy of a robot can be degraded considerably if certain strict conditions on rigidity are not maintained. But in order to construct the links of the robot for rigidity, one must pay the price in terms of weight. Heavy links require large actuators to move them. Large actuators will in turn consume more power. This will affect the payload capacity and the speed of operation adversely. Also, the resulting robot will make it very difficult to transport it from one location to another. This problem is especially critical for space applications. This paper addresses such problems related to space applications of robots.

All problems that stem from the heavy weight of the links can be overcome by relaxing the rigidity constraint. This would enable the robot designer to construct the links of nonconventional robot structurals such as aluminum alloys and composite materials. The resulting slender, lightweight links would be strong in compression, but comparatively weaker in bending. Effective control of such a flexible manipulator is of great import in space. But before the flexible manipulator can be controlled, its dynamics have to be investigated in detail.

Several researchers have modeled a flexible link and studied its dynamics.[1-6] There are several methods in use. They vary from experimental[1] through analytical[2] to finite-element modeling.[4-5] It is obvious that the manipulator dynamics strongly influence the position control and accuracy. Detailed studies have shown that the first two or three elastic modes play a significant role in the dynamic behavior of a flexible link.

The position control of a flexible link poses a challenge to the control system designer. General proportional-integral-derivative (PID) control improves the dynamic performance, but the error in the positioning of the manipulator endpoint is still quite large. This is due to the nonlinear nature of the process. The link-inertias change continuously with time and position. The performance of the position control of a robot may be improved by the use of adaptive control systems. The advantages of adaptive control systems have been described in many papers and books.[7-14] The most promising of such systems is model reference adaptive control (MRAC).[14-19] This method is stable over a broad range of applications and gives good results if applied carefully. It allows us to overcome the influence of nonlinearities. MRAC has reduced sensitivity to payload, e.g., the dynamic performance remains unaffected by variations in the payload carried by the robot.

Problem Formulation

The problem addressed here was the development of an effective method for controlling flexible manipulators. Dynamics of the manipulator have been included in the designing process of its controller. Regular PID control did not give satisfactory results; therefore, an adaptive control system was investigated. MRAC has been chosen to control the position of the manipulator. The objective is to design an adaptive control that will perform better than genuine PID control, i.e., the position errors will be smaller with better vibration characteristics than with PID control.

Mathematical Model

An accurate mathematical modeling of a flexible manipulator is an essential prerequisite for its position and vibration control. Two of the popular methods used for modeling are those based on finite-element methods and those based on modal expansion methods. The latter technique has been demonstrated in this paper to model a flexible manipulator.

The single-link flexible manipulator is modeled as a pinned-free beam of uniform distribution, as illustrated in Fig. 1. The following assumptions are made:

1) The link is uniform along is longitudinal direction, both in its mass distribution and elastic properties.

2) The transverse shear stresses and the moment of inertia with respect to elastic deformation are negligible, i.e., Euler's beam theory is applicable.

3) The elastic deformation of the link is very small and, hence, all of the second- or higher-order terms in the equation for the elastic deformation are negligible.

4) The change in potential energy of gravity due to elastic deformation of the link is negligible.

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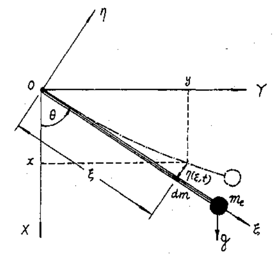

Fig. 1 Single-link flexible manipulator.

With respect to Fig. 1, XOY is the vertical plane, and $\xi O \eta$ is the coordinate frame with the $O\xi$-axis aligned along the neutral axis of the link. The angular displacement of the neutral axis of the link from the OX axis is denoted by θ, and m_e is the payload mass at the end-effector.

Considering an elemental mass dm along the link at a distance ξ, we denote its elastic displacement from the rigid position by $\eta(\xi,t)$. We can then make the following coordinate transformation as

$$\left.\begin{aligned} x &= \xi \cos\theta - \eta \sin\theta \\ y &= \xi \sin\theta + \eta \cos\theta \end{aligned}\right\} \tag{1}$$

Differentiating Eq. (1) with respect to time, we get

$$\dot{x} = -(\xi \sin\theta + \eta \cos\theta)\dot{\theta} - \dot{\eta} - \dot{\eta} \sin\theta$$

$$\dot{y} = +(\xi \cos\theta - \eta \sin\theta)\dot{\theta} + \dot{\eta} \cos\theta$$

Therefore, the Cartesian velocity v of the elemental mass becomes

$$\begin{aligned} v^2 &= \dot{x}^2 + \dot{y}^2 \\ &= (\xi^2 + \eta^2)\dot{\theta}^2 + \dot{\eta}^2 + 2\dot{\theta}\xi\dot{\eta} \end{aligned} \tag{2}$$

The kinetic energy of the elemental mass becomes

$$\begin{aligned} dK &= 0.5v^2 \, dm \\ &= 0.5[(\xi^2 + \eta^2)\dot{\theta}^2 + \dot{\eta}^2 + 2\dot{\theta}\xi\dot{\eta}] \, dm \end{aligned} \tag{3}$$

Integrating Eq. (3) along the whole length of the link, we get the total kinetic energy of the system as

$$\begin{aligned} K = {} & 0.5\int_0^L [(\xi^2 + \eta_2)\dot{\theta}^2 + \dot{\eta}^2 + 2\dot{\theta}\xi\dot{\eta}] \, dm \\ & + 0.5m_e[(L^2 + \eta_L{}^2)\dot{\theta}^2 + \dot{\eta}_L{}^2 + 2L\dot{\theta}\dot{\eta}_L] \end{aligned} \tag{4}$$

where $\eta_L = \eta(\xi,t)_{\xi=L}$, and $\dot{\eta}_L = \dot{\eta}(\xi,t)_{\xi=L}$. Using the assumed mode method,[6] the elastic deflection of the link can be written as a series of modal shape functions of a uniform cantilever beam as

$$\eta(\xi,t) = \sum_{i=1}^{n} e_i(t)\phi_i(\xi) = [\phi(\xi)]^T[z(t)] \tag{5}$$

where

$e_i(t)$ = generalized coordinates of elastic deflection

$\phi_i(\xi)$ = ith modal shape function

$= \cosh(B_i\xi) - \cos(B_i\xi) - r_i[\sinh(B_i\xi) - \sin(B_i\xi)]$

$r_i = [\cosh(B_i) + \cos(B_i)]/[\sinh(B_i) + \sin(B_i)]$ and

$B_i = 1.875, 4.694, 7.855, \ldots$

$$[\phi(\xi)] = [\phi_1(\xi), \phi_2(\xi), \ldots, \phi_n(\xi)]^T$$

$$[z(t)] = [e_1(t), e_2(t), \ldots, e_n(t)]^T$$

$$n = 1, 2, 3, \ldots$$

Now, substituting Eq. (5) into Eq. (4) gives

$$\begin{aligned} K = {} & 0.5\left\{\int_0^L \xi^2 \, dm + m_e L^2\right\}\dot{\theta}^2 \\ & + 0.5[z]^T\left\{\int_0^L [\phi][\phi]^T \, dm + m_e[\phi(L)][\phi(L)]^T\right\}[z]\dot{\theta}^2 \\ & + 0.5[\dot{z}]^T\left\{\int_0^L [\phi][\phi]^T \, dm + m_e[\phi(L)][\phi(L)]^T\right\}[\dot{z}] \\ & + \dot{\theta}\left\{\int_0^L \xi[\phi]^T \, dm + m_e L[\phi(L)]^T\right\}[\dot{z}] \end{aligned}$$

i.e.,

$$K = 0.5I_\theta\dot{\theta}^2 + 0.5\dot{\theta}^2[z]^T[I_\phi][z] + 0.5[\dot{z}]^T[I_\phi][\dot{z}] + \dot{\theta}[I_{\theta\phi}]^T[\dot{z}] \tag{6}$$

with

$$I_\theta = \rho A L^3/3 + m_e L^2$$

$$[I_\phi] = \rho A\int_0^L [\phi][\phi]^T \, d_\xi + m_e[\phi(L)][\phi(L)]^T$$

$$[I_{\theta\phi}]^T = \rho A\int_0^L \xi[\phi]^T \, d\xi + m_e L[\phi(L)]^T$$

ρ = mass density of the material of the link

A = area of cross section of the link

m_e = payload mass

In order to apply the Lagrangian method for the dynamic equations, the potential energy of the system must also be determined. The potential energy of elastic deformation of the link is given by Euler's beam theory as

$$\begin{aligned} U_e &= 0.5\int_0^L EI\left(\frac{\partial^2\eta}{\partial\xi^2}\right)^2 d\xi \\ &= 0.5\int_0^L EI[z]^T[\phi''(\xi)][\phi''(\xi)]^T[z] \, d\xi \end{aligned} \tag{7}$$

i.e.,

$$U_e = 0.5[z]^T[K^*][z]$$

where

$$[K^*] = \int_0^L EI[\phi''(\xi)][\phi''(\xi)]^T \, d\xi \tag{8}$$

and

$$[\phi'(\xi)] = \frac{\partial\phi}{\partial\xi}, \qquad [\phi''(\xi)] = \frac{\partial^2\phi}{\partial\xi^2}$$

The potential energy due to gravity can be written as

$$U_g = -\int_0^L xg \, \mathrm{d}m - x_L m_e g$$
$$= \int_0^L (\xi \cos\theta - \eta \sin\theta)\rho A \, \mathrm{d}\xi - g$$
$$\times (L \cos\theta - \eta_L \sin\theta) m_e \tag{9}$$

i.e.,

$$U_g = -S_0 \cos\theta + \sin\theta [S_\phi]^T [z] \tag{10}$$

where

$$S_0 = 0.5 \,\rho g A L^2 + m_e g L \tag{11a}$$

$$[S_\phi]^T = \rho g A \int_0^L [\phi]^T \, \mathrm{d}\xi + m_e g [\phi(L)]^T \tag{11b}$$

The Lagrangian function of the system given by

$$L = K - U_e - U_g$$

for the manipulator system then becomes

$$L = 0.5 I_\theta \dot\theta^2 + 0.5[z]^T [I_\phi][z]\dot\theta^2 + 0.5[\dot z]^T [\dot z]$$
$$[I_\phi]\,[\dot z] + \dot\theta [I_{\theta\phi}] T [\dot z]$$
$$- 0.5[z]^T [K^*][z] + S_0 \cos\theta - \sin\theta [S_\phi]^T [z] \tag{12}$$

The equations of motion can then be written as

$$\tau = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot\theta}\right) - \frac{\partial L}{\partial \theta}$$
$$= I_\theta \ddot\theta + [I_{\theta\phi}]^T [\ddot z] + 2[z]^T [I_\phi][\dot z]\dot\theta + [z]^T [I_\phi][z]\ddot\theta$$
$$+ S_0 \sin\theta + \cos\theta [S_\phi]^T [z] \tag{13a}$$

and

$$F = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial [\dot z]}\right) - \frac{\partial L}{\partial [z]} \tag{13b}$$

$$0 = \ddot\theta [i_{\theta\phi}] + [I_\phi][\ddot z] + [K^*][z] - \dot\theta^2 [I_\phi][z] + \sin\theta [S_\phi] \tag{13c}$$

Using assumptions (3) and (4), Eqs. (13) can be simplified to the nonlinear form:

$$[M][\ddot X] + [K][X] = [F] \tag{14}$$

where

$$[X] = \{\theta [z]^T\}^T$$

$$[M] = \begin{bmatrix} I_\theta & [I_{\theta\phi}]^T \\ [I_{\theta\phi}] & [I_\phi] \end{bmatrix}$$

$$[K] = \begin{bmatrix} 0 & [0] \\ [0] & [K^*] \end{bmatrix}$$

$$[F] = \{(\tau - S_0 \sin\theta)\ [0]\}^T$$

and where τ = actuator torque at the pinned end.

To include the damping effects of the manipulator in the equation of motion, we further assume that the damping is viscous of the form

$$T_d = -C_\theta \dot\theta \tag{15a}$$

$$C_i = 2\zeta_i \sqrt{m_i K_i} \tag{15b}$$

where

C_θ = damping coefficient in the joint

ζ_i = damping ratio of the ith mode of the link

M_i = generalized mass of the ith mode

K_i = generalized stiffness of the ith mode

Eq. (14) can then be rewritten as

$$[M][\ddot X] + [C][\dot X] + [K][X] = [F] \tag{16}$$

where

$$[C] = \mathrm{diag}\,(C_\theta, C_1, C_2, \ldots C_{n-1})$$

Dynamic Model

The vibration analysis of a single-link flexible manipulator described by Eq. (16) is composed of two main parts: modal expansion analysis and the transient response analysis.

Modal Expansion of the Manipulator

The dynamics of the flexible manipulator are coupled, in the sense that the rigid and flexible modes of such a manipulator are closely interrelated. Further, such a distributed mass model represents an infinite dimensional system. The technique of modal expansion allows us to develop a finite-dimensional model for the preceding system. This means that in Eq. (5), $\eta(\xi,t)$ can be expressed as a finite sum of the first few modes (usually 3-5). The first three mode shapes of the flexible manipulator are shown in Fig. 2.

The link of the flexible manipulator used in this study is made of steel and has a hollow circular cross section with the following properties.

length L = 1.0 m
density ρ = 7806.0 kg/m^3
outer radius r_o = 0.03 m
inner radius r_i = 0.01 m
Young's modulus E = 2.07 × 10^{11} N/m^2
joint damping = 0.1 N-ms/rad
link mass = 19.62 kg

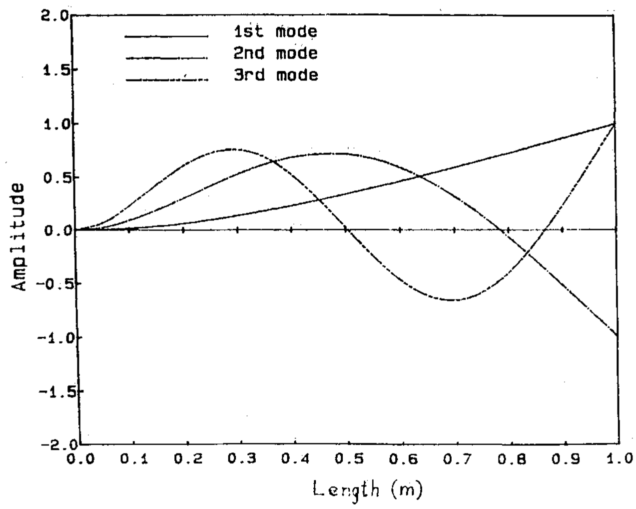

Fig. 2 Modal shapes of the manipulator.

Table 1 Eigenvalues and eigenvectors

	1st mode	2nd mode	3rd mode
Eigenvalues (rad/s)	0.00	994.36	5522.00
Eigenvectors	0.29438	1.0674	−5.3733
	0.00	−0.54736	3.1190
	0.00	0.089613	0.48503

A computer program was developed to perform the vibration analysis of the flexible manipulator. The Gaussian (five points) integration technique was used for numerical integration. The undamped vibration modes of the manipulator were calculated by using the standard quadratic linear (QL) method and are listed in Table 1.

Other parameters used in the program are

desired angular position = 0.5
desired angular velocity = 0.5 rad/s
desired period = 1.333 s
length of link = 1 m
payload mass = 5 kg

Transient Vibration of the Manipulator

For the transient response analysis of the system, described by Eq. (16), it is more convenient to transform it into a state-space form as

$$\begin{Bmatrix} \ddot{X} \\ \dot{X} \end{Bmatrix} = \begin{bmatrix} -[M]^{-1}[C] & -[M]^{-1}[K] \\ [I] & [0] \end{bmatrix} \begin{Bmatrix} \dot{X} \\ X \end{Bmatrix} + \begin{Bmatrix} [M]^{-1}[F] \\ 0 \end{Bmatrix} \tag{17}$$

To solve the above first-order differential equation, the Runge-Kutta numerical integration technique has been used.

In order to move the manipulator arm to a desired angular position while following a desired trajectory in terms of position and velocity, the actuator torque needs to be calculated. A simple feedback control law of the form

$$\tau = -K(\dot{\theta} - \dot{\theta}_d) + S_0 \sin\theta \tag{18}$$

is used where $\dot{\theta}_d$ is the desired angular velocity of the manipulator and K is the derivative gain or gain of the velocity feedback control equal to 100.0

It must be noted that the second term $S_0 \sin\theta$ in Eq. (18) is a gravity compensation term. The transient response of the manipulator in terms of angular position and angular velocity is shown in Figs. 3 and 4 for various payload masses. Figure 5 shows the transient response of the first mode of the manipulator link. It was found in subsequent experiments that the second mode is considerably smaller than the first, and the third even more so. Therefore, it is adequate to consider the first two elastic modes and still be assured of good accuracy.

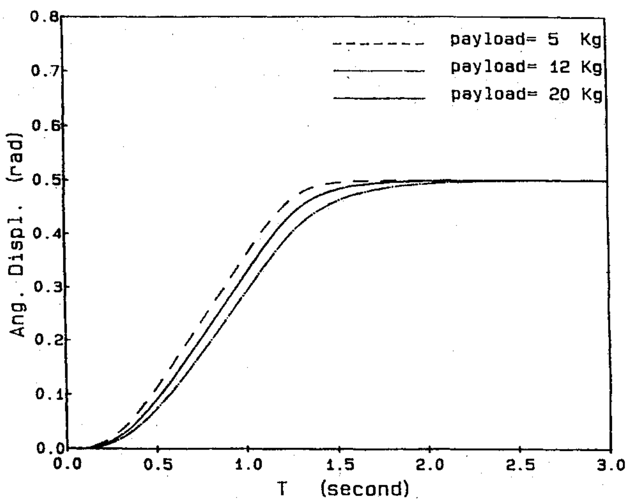

Fig. 3 Transient response of the manipulator.

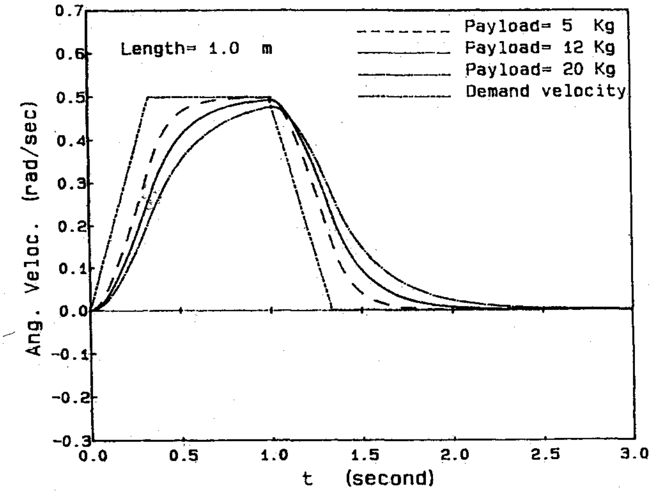

Fig. 4 Transient response of the manipulator.

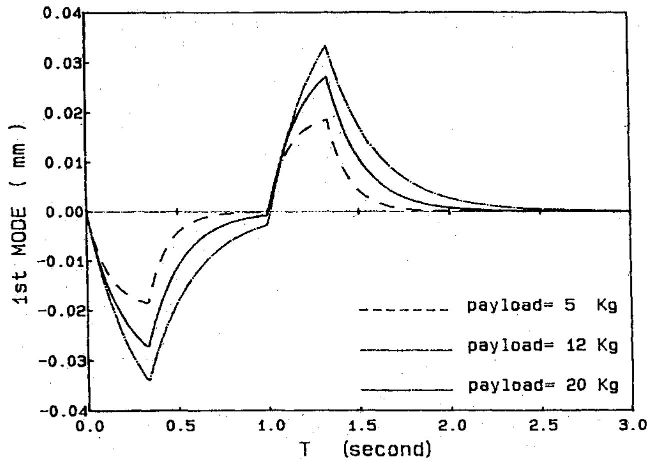

Fig. 5 Transient response of the manipulator.

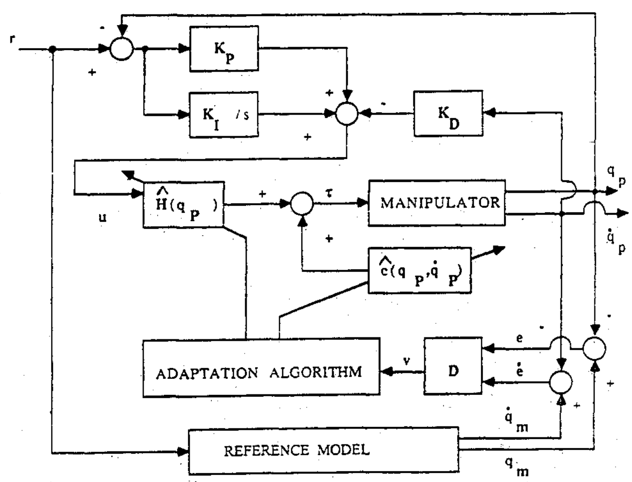

Fig. 6 Block diagram of model reference adaptive control.

Model Reference Adaptive Control

The manipulator together with the control system is depicted in the form of a block diagram in Fig. 6. This particular MRAC scheme was first suggested by Asare and Wilson.[17] Another popular MRAC scheme can be found in Ref. 19. A mathematically defined reference model is operated in parallel with the manipulator itself. Both the model and the manipulator are essentially controlled by a PID controller with position and velocity feedback. At the beginning, the manipulator is assumed to be at its rest position, i.e., its link is aligned in the outstretched horizontal position. In this position $\theta_i = 0$ rad, where $i = 1, 2, 3$. Now, θ_1 is the joint angle relating to the rigid mode, and θ_2 and θ_3 are angles relating to the first two flexible modes, i.e., $\theta_2 = \eta_1(L,t)/L$, and $\theta_3 = \eta_2(L,t)/L$.

The reference model, which is represented by a double integrator together with its PID controller, forms a third-order system, henceforth referred to as the ideal system. The output vector of this ideal system, viz., q_m and $\dot{q}_m$, are the joint angles and joint rates, respectively, of the reference model. Similarly, the joint angles and rates of the manipulator itself are represented by q_p and $\dot{q}_p$, respectively. This q_p and $\dot{q}_p$ are compared with q_m and $\dot{q}_m$, respectively, to yield the error vectors representing the differences between them. These error vectors are used in the adaptation algorithm to modify the plant parameters. Asare and Wilson[17] have evaluated the relative merits of two types of adaptive control, i.e., one in which the plant parameters are adapted and the other in which the PID gains are adapted. They concluded that the former method yields a more stable and robust sytem than the latter. The authors arrived at the same conclusion separately. It is for this reason that the authors have chosen the former method here.

Using the Lagrangian method of derivation, the dynamics equations of the flexible manipulator can be obtained in the form

$$\tau(t) = H(q_p).\ddot{q}_p + C(q_p,\dot{q}_p) \tag{19}$$

where q_p, $\dot{q}_p$, and $\ddot{q}_p$ are the (3×1) generalized position, velocity, and acceleration vectors, respectively. $H(q_p)$ is the (3×3) generalized inertia matrix of the manipulator (GIM), $C(q_p, \dot{q}_p)$ is the (3×1) vector representing nonlinearities due to the dynamic coupling between the modes, and $\tau(t)$ is the (3×1) joint-torque input vector.

The nonlinearity compensation and decoupling control are achieved by

$$\tau(t) = \hat{H}(q_p).u(t) + \hat{C}(q_p, \dot{q}_p) \tag{20}$$

where $\hat{H}(.)$ and $\hat{C}(.,.)$ are estimated values of $H(.)$ and $C(.,.)$. Next, the PID controller is applied to both the manipulator and the reference model:

$$u(t) = K_P(r - q_p) + K_I\int_0^t(r - q_p)\,\mathrm{d}t - K_D\,\dot{q}_p \tag{21}$$

$$\ddot{q}_m = K_P(r - q_m) + K_I\int_0^t(r - q_m)\,\mathrm{d}t - K_D\,\dot{q}_m \tag{22}$$

where K_P, K_I, and K_D are the PID controller gains, and q_p and q_m are the generalized variables of the manipulator and reference model, respectively.

The manipulator itself can be represented by

$$\dot{q}_p(t) = \int \ddot{q}_p(t)\,\mathrm{d}t \tag{23a}$$

$$q_p(t) = \int \dot{q}_p(t)\,\mathrm{d}t \tag{23b}$$

where $\ddot{q}_p(t)$ is given by

$$\ddot{q}_p(t) = [H(q_p)]^{-1}\,[\hat{H}(q_p)u(t) + (\hat{C} - C)] \tag{24}$$

The reference model can be represented by

$$\dot{q}_m(t) = \int \ddot{q}_m(t)\,\mathrm{d}t \tag{25a}$$

$$q_m(t) = \int \dot{q}_m(t)\,\mathrm{d}t \tag{25b}$$

The position and velocity error vectors are defined as

$$e(t) = q_m - q_p \tag{26a}$$

$$\dot{e}(t) = \dot{q}_m - \dot{q}_p \tag{26b}$$

Defining a composite error vector $\nu(t)$ as

$$\nu(t) = \nu_P + \nu_D \tag{27}$$

where $\nu_P = D_p e_p$ and $\nu_D = D_v\dot{e}_p$.

The adaptation algorithm (28) for the three-DOF manipulator is then given by

$$\dot{\hat{h}}_{ii}\alpha_{ii}v_iu_i, \qquad i = 1, 2, 3 \tag{28a}$$

$$\dot{\hat{h}}_{ij} = \alpha_{ij}[v_iu_j + v_ju_i], \quad i = 1,2, \quad j = i+1, \ldots, 3 \tag{28b}$$

$$\dot{\hat{C}}_{ij}{}^{(1)} = \beta_{ij}{}^{(1)}[2\dot{q}_{pi}\dot{q}_{pj}v_i - \dot{q}_{pi}{}^2v_j], \quad i = 1, \quad j = 2, 3 \tag{28c}$$

$$\dot{\hat{C}}_{ii}{}^{(k)} = \beta_{ii}{}^{(k)}[\dot{q}_{pi}{}^2v_k], \quad k = 1, 2, \quad i = k+1, \ldots, 3 \tag{28d}$$

$$\dot{\hat{C}}_{23}{}^{(1)} = \beta_{23}{}^{(1)}[2\dot{q}_{p2}\dot{q}_{p3}v_1] \tag{28e}$$

$$\dot{\hat{C}}_{13}{}^{(2)} = 2\beta_{13}{}^{(2)}[\dot{q}_{p1}\dot{q}_{p3}v_2 - \dot{q}_{p1}\dot{q}_{p3}v_3] \tag{28f}$$

$$\dot{\hat{C}}_{23}{}^{(2)} = 2\beta_{23}{}^{(2)}[\dot{q}_{p1}\dot{q}_{p2}v_2 - \dot{q}_{p2}{}^2v_3] \tag{28g}$$

where h_{ij} represents the $(i$-$j)$th term of the $H(q_p)$ matrix, $C_{ij}{}^{(k)}$ that of the matrix form of the $C(q_p, \dot{q}_p)$ vector, and α_{ij} and $\beta_{ij}{}^{(k)}$ their corresponding adaptation gains. The actual values of $H(q_p)$ and $C(q_p, \dot{q}_p)$ are given by the dynamics equations.

Simulation Results

The dynamic modeling with an appropriate control system has been simulated on the Apollo workstation. The simulation program has been written in the language "C." The results have been shown in Figs. 7–12. Figure 7 presents the angular position response to a step input in the form of 0.5 rad final angular position of the link. It can be seen that the response of the system without adaptation is different from that with adaptation. The initial response of the unadapted system is slower, and there is considerable overshoot. Also, it takes longer to reach the final position.

Figure 8 shows a similar pattern of response for an impulse input added to the previous step input. Again, the system with adaptation shows better dynamic response.

Figure 9 displays a free response of the link tip to an impulse input. The link-tip deflection plotted against time has been computed using Eq. (5). In this case, the first two flexible modes of vibration have been considered. The response shows two peaks of large magnitude. These peaks are the result of the coupling between the rigid and flexible modes.

Since the free response maximum is -1.5 mm, it is desirable to have dynamic compensators in order to reduce the error. A simple proportional-derivative (PD) controller has been used

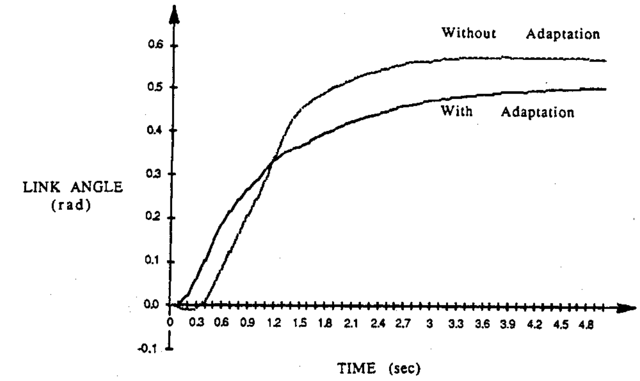

Fig. 7 Step response of joint angle.

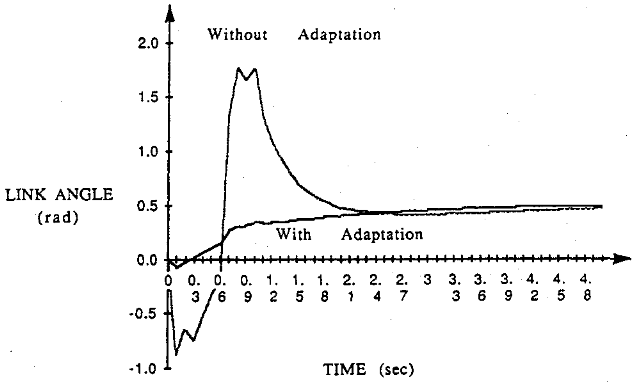

Fig. 8 Impulse response of joint angle.

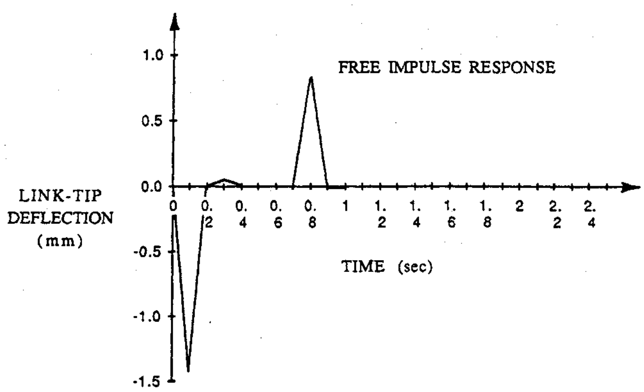

Fig. 9 Free impulse response of the flexible manipulator.

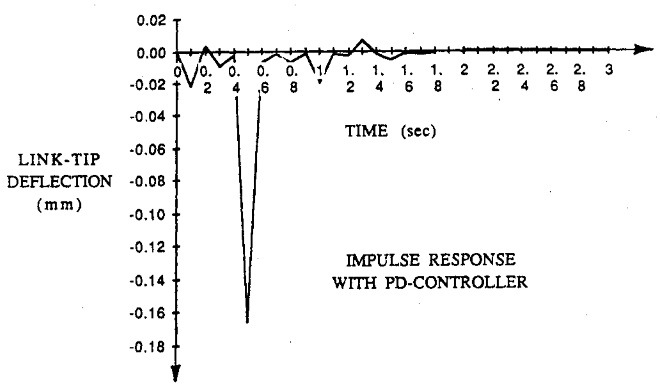

Fig. 10 Impulse response with proportional-derivative controller.

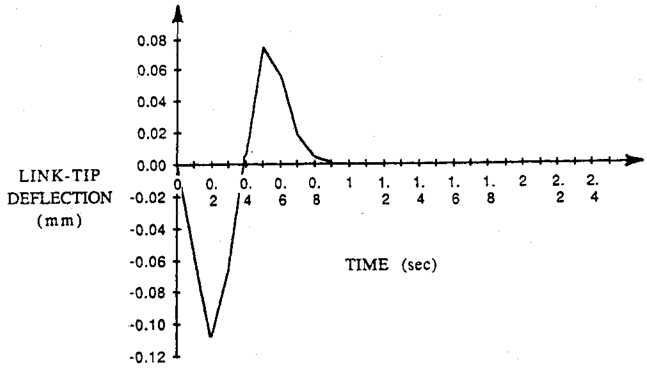

Fig. 11 Flexible manipulator response with model reference adaptive control.

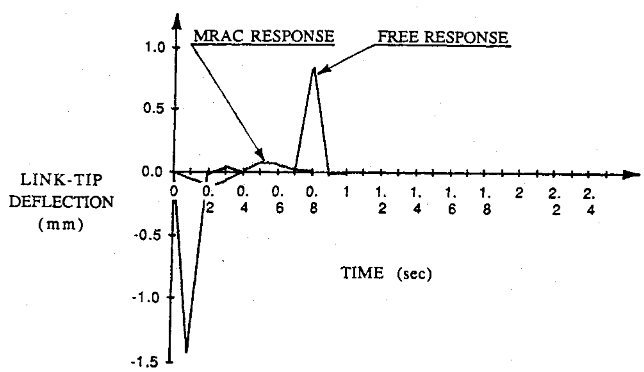

Fig. 12 Comparison between free and adaptive responses.

to achieve this. Figure 10 shows an improvement in link-tip position control. The maximum error has been reduced to -0.17 mm. This is a significant improvement in position control; however, the single maximum needs further smoothing and reduction to be of any value for precision manipulators.

Therefore, an adaptive control technique based on model-reference adaptive control has been applied to improve further the response characteristics of the single-link flexible manipulator. The structure of the control system has been described in the Model Reference Adaptive Control section. Just as the resultant deflection of the flexible link is given by the sum of the deflections due to the individual modes, the resultant joint torque to be applied by the actuator, too, can be computed as the sum of the torques required to control the individual modes. Then, $\tau = \tau_1 + \tau_2 + \tau_3$.

The manipulator respose with MRAC has been plotted in Fig. 11. Further reduction of the error (from -0.17 mm top -0.11 mm) can be seen. Also, the settling time is halved. The comparison between free and MRAC responses has been shown in Fig. 12. Fig. 13 shows the convergence characteristics of the actual and estimated values of one of the nonlinear terms in matrix C.

Conclusions

1) Dynamic modeling of a flexible manipulator with distributed mass by the method of modal expansion is considered to be accurate, and in inclusion of flexibility of the link makes its dynamic equations strongly nonlinear.

2) The discontinuities in the manipulator response caused by coupling between the rigid and flexible modes can be eliminated by appropriate control methods.

3) A simple proportional-derivative controller is able to improve the dynamic response of the manipulator, whereas a

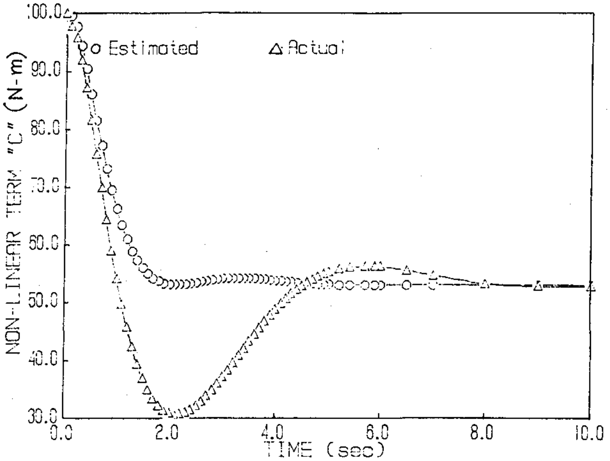

Fig. 13 Convergence characterstics on nonlinear term C.

model reference adaptive controller further reduces the positioning error of the end-effector and shortens the settling tiem, thus showing that an adaptive control system gives best results in applications where precision control of flexible manipulators are required.

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